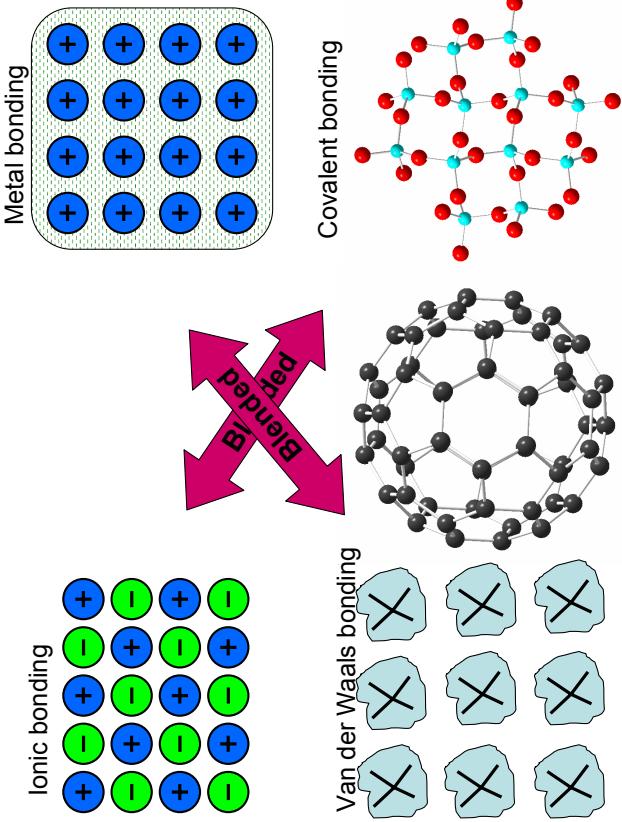
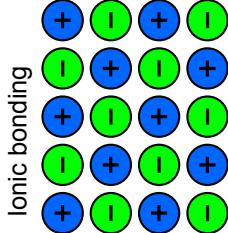
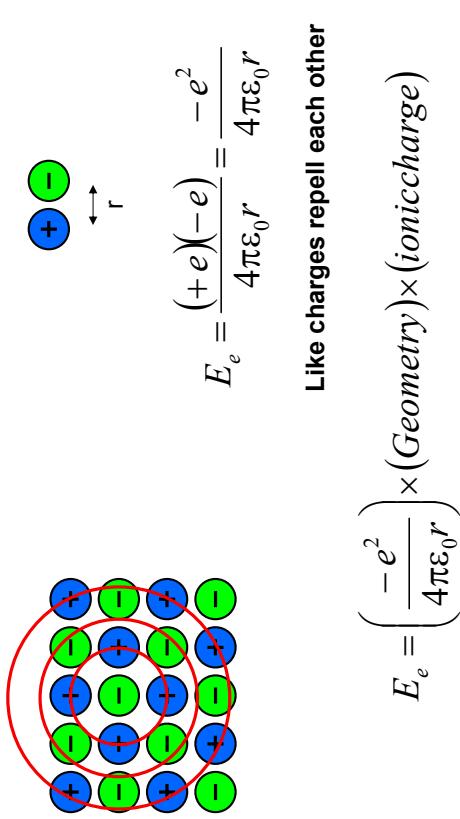


Crystalline materials show a range of different bond types:

Kap. 2 Chemical bonding



Madelung energy



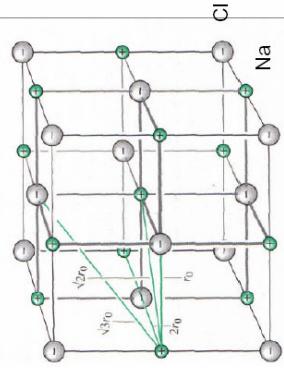
Ionic structures – Lattice energy - NaCl

Na^+ is surrounded by 6 Cl^-

$$V = -\frac{6(Z_+)(Z_-)e^2}{\sqrt{2r}}$$

Na^+ is surrounded by 12 next neighbour Na^+ at $\sqrt{2}r$

$$V = -\frac{12(Z_+)(Z_-)e^2}{\sqrt{2r}}$$



Na^+ is surrounded again by 8 Cl^- at $\sqrt{3}r$

$$V = -\frac{8(Z_+)(Z_-)e^2}{\sqrt{3}r}$$

$$V = -\frac{(Z_+)(Z_-)e^2}{r} \left(6 - \frac{12}{\sqrt{2}} + \frac{8}{\sqrt{3}} - \frac{6}{\sqrt{4}} + \dots \right)$$

Madelung energy

$$E_e = N_A \left(\frac{e^2}{4\pi\epsilon_0 r} \right) \alpha (Z_M Z_X)^2 \frac{m+n}{2}$$

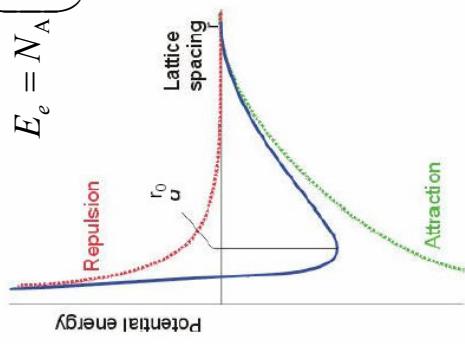
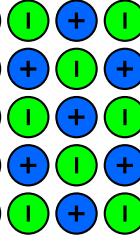
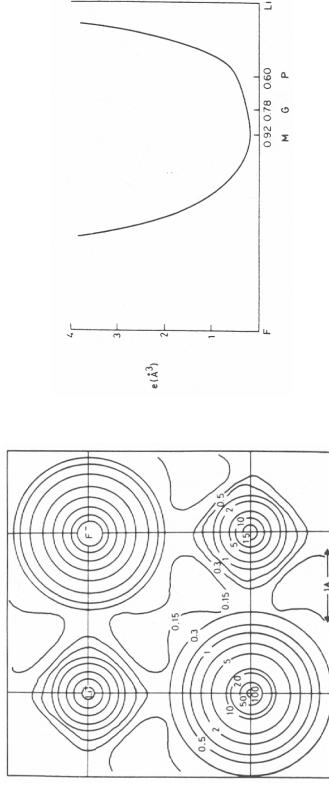


Table 2.1 Reduced Madelung constants, α					
Structure	Formula	Example	α		
Halite	M^+X^-	NaCl	1.748		
Caesium chloride	M^+X^-	CsCl	1.763		
Sphalerite	M^2+X^{2-}	ZnS	1.638		
Wurtzite	M^2+X^{2-}	ZnO	1.641		
Fluorite	$M^2+X_2^-$	CaF ₂	1.68		
Rutile	$M^4+X_2^-$	TiO ₂	1.60		

Ions also show repulsive forces when they are too close!

Ionic bonding

The ionic radii from Pauling and Goldschmidt have been revised due to information from present high-quality X-ray diffraction work.



Ionic radii

Non-directional
High coordination number as possible
Charged, non-compressible, non-polarizable spheres

Purely ionic binding rarely occurs,
a net charge of more than +1 are unlikely

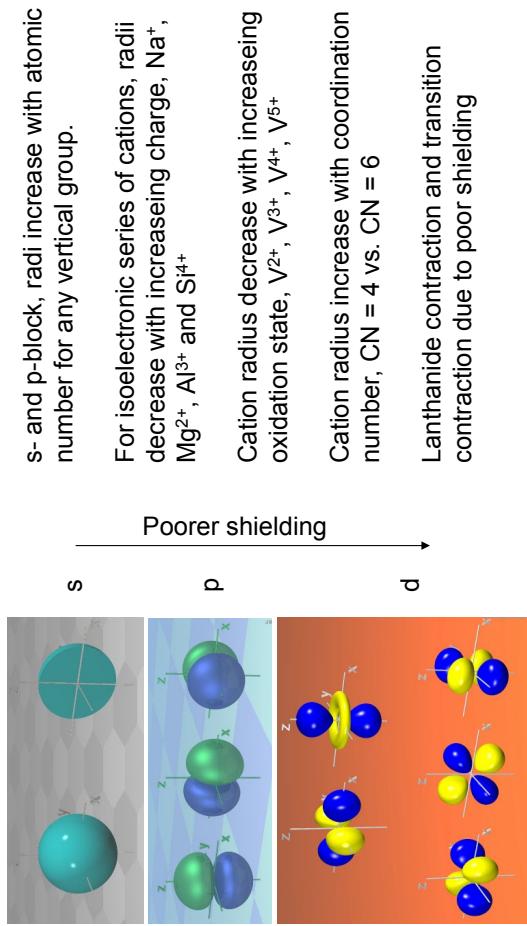
Useful as starting point to visualize structures

For visualization, one needs a set of ionic radii

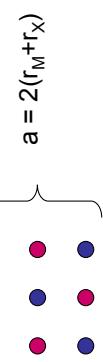
- a) Ions are essentially spherical
- b) Ions are composed of a central core with most of the electrons and an outer sphere with very little electron density.
- c) Assignment of radii is difficult

Ionic radii

The size of an ion is dependent on how the outer orbitals shield the charge from the nucleus.



Ionic radii



For isolectronic series of cations, radii decrease with increasing charge, Na^+ , Mg^{2+} , Al^{3+} , and Si^{4+}

Cation radius decrease with increasing oxidation state, V^{2+} , V^{3+} , V^{4+} , V^{5+}

Cation radius increase with coordination number, CN = 4 vs. CN = 6

Lanthanide contraction and transition contraction due to poor shielding

Consistent set of values

The values are dependent on:

- Coordination number
- Type of coordination polyhedra
- Oxidation number

Metallic oxides

Sett of radii dependent on type of orbitals used??

Ionic structures – general principles

1. Ions are in fact charged, elastic and polarizable spheres

2. Geometrical arrangement

contraction:
repulsion:
anion – cation
anion – anion
cation – cation

3. As high coordination number as possible to maximize net electrostatic attraction. **Depends on the ratio of ion radii**.

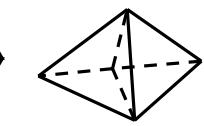
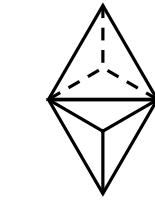
4. Next neighbour ions are as far as possible. → Maximization of volume!

5. Local electroneutrality. Valence = Σ electrostatic bond strength

Force between two ions:

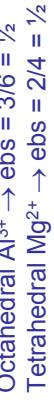
$$F = \frac{(Z_+ e)(Z_- e)}{r^2}$$

Electrostatic bond strength:
The charge is divided among the number of bonds.

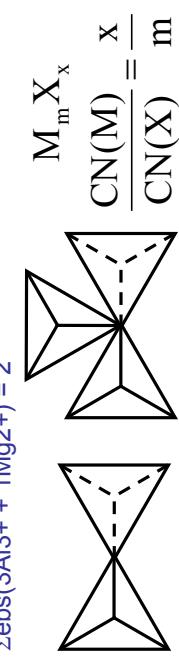


$$\sum \frac{m}{n} = x$$

The sum of ebs on an ion must balance the charge



→ Oxygen tetrahedrally surrounded by 3 Al and 1 Mg:



SiO₂ ?

$$\frac{M_m X_x}{CN(M)} = \frac{x}{CN(X)}$$

Ionic structures – bond strength

Ionic structures – bond strength

Example:

NaCl type structure, Na⁺ in octahedra holes → CN(Na⁺) = 6

CN (Cl⁻)?

CN (Cl⁻) = 1/1*6 = 6

CaF₂ type structure, F⁻ in tetrahedra holes → CN(F⁻) = 4

CN (O²⁻)?

CN (O²⁻) = 1/2*6 = 3

Rutile type structure (TiO₂); CN(Ti⁴⁺) = 6

CN (O²⁻)?

CN (O²⁻) = 2/1*4 = 8

Table 2.1 Electrostatic bond strengths of some cations

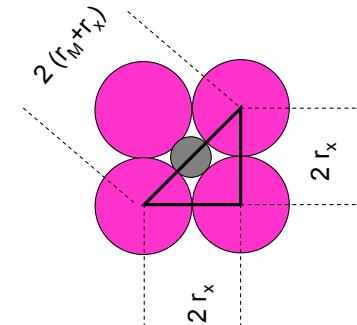
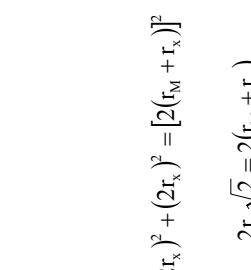
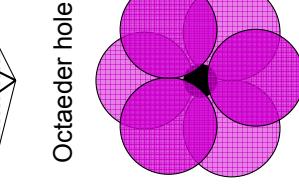
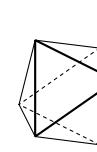
Cation	Coordination number(s)	e_b/s
Li ⁺	4, 6	1/4
Na ⁺	6, 8	1/6
Be ²⁺	3, 4	2/3
Mg ²⁺	4, 6	1/3
Ca ²⁺	8	1/2
Zn ²⁺	4	1/2
Al ³⁺	4, 6	1/4
Cr ³⁺	6	1/4
Si ⁴⁺	4	1
Ge ⁴⁺	4, 6	1/3
Ti ⁴⁺	6	1/3
Th ⁴⁺	8	1/4

Table 2.2 Allowed and unallowed combinations of corner-sharing oxide polyhedra

Allowed	Example	Unallowed
2SiO ₄ tet.	Silica	> 2SiO ₄ tet.
1MgO ₄ tet. + 3Al ₂ O ₆ oct.	Olivine	3Al ₂ O ₆ oct.
1SiO ₄ tet. + 3MgO ₆ oct.	Li ₂ O	1SiO ₄ tet. + 2Al ₂ O ₆ oct.
8Li ₂ Otet.	Peroxskite	4TiO ₆ oct.
2TiO ₆ oct. + 4CaO ₁₂ dod.	Rutile	3TiO ₆ oct.

Ionic structures – Radius ratio

Ideally ions surround themselves with as many ions of the opposite charge as possible. The ions must be in contact → CN depends on the radius ratios.



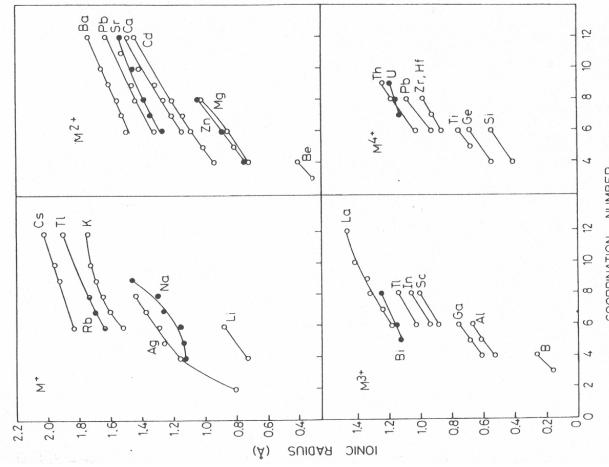
$$(2r_x)^2 + (2r_x)^2 = [2(r_M + r_x)]^2$$

$$2r_x\sqrt{2} = 2(r_M + r_x)$$

$$\frac{r_M}{r_x} = \sqrt{2} - 1 = 0.414$$

Ionic structures – Radius ratio

Table 2.4 Structures and radius ratios of oxides, MO₂



* Since cation radii vary with CN, Fig. 2.3, radius ratios may be calculated for different CNs. The CNs used here are shown in parentheses. Calculations are based on $r_{O^{2-}} = 1.26 \text{ \AA}$.

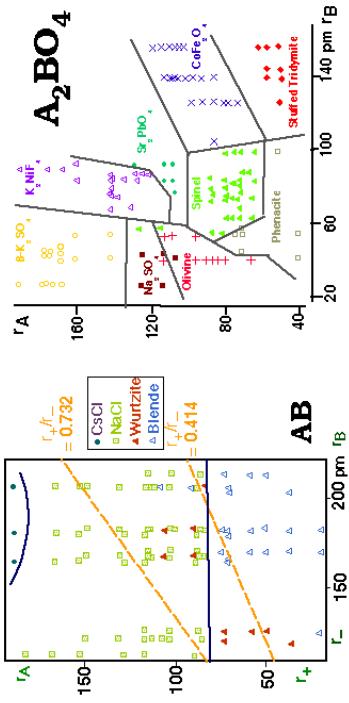
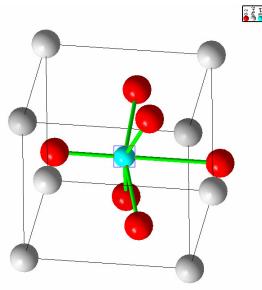
Ionic structures – Radius ratio, borderline

Ionic structures – Radius ratio

GeO_2 has polymorphs where Ge may have CN = 4 or 6

Al^{3+} may be found in octahedral environments for spinel MgAl_2O_4 , but also in tetrahedras for silicates.

Ti in PbTiO_3 is slightly too small and may rattle. Displaced by 0.2 Å.



Ionic structures – Lattice energy



Sublimation energy

$$F = \frac{(Z_+)(Z_-)e^2}{r^2}$$

Force between two ions

$$V = \int_{\infty}^r F dr = -\frac{(Z_+)(Z_-)e^2}{r}$$

Coulombic potential energy

$$V = \frac{B}{r^n} \quad n = 5 \dots 12$$

Short-range repulsive forces

Ionic structures – Lattice energy

Attractive electrostatic forces

$$U = \frac{A e^2 Z_+ Z_- N}{r} + B N e^{-r/p} - C N r^{-6} + 2.25 N h v_{\text{Omax}}$$

Van der Waals attractive forces

Born repulsive term

Zero point energy

Typical values for the terms, in kJ mol^{-1} , are (from Greenwood):

Substance	$NA^2 Z_+ Z_- r^{-1}$	$NBe^{-r/p}$	NCr^{-6}	$2.25 N h v_{\text{Omax}}$	U
NaCl	-859.4	98.6	-12.1	7.1	-755.8
MgO	-4631	698	-6.3	18.4	-3921

The Kapustinskii equation

The Kapustinskii equation

Kapustinskii found that if the Madelung constant for a given structure is divided by the number of ions in one formula unit (η) the resulting values are almost constant:

Structure	Madelung const.(A)	A/v	Coordination
CsCl	1.763	0.88	8:8
NaCl	1.748	0.87	6:6
CaF ₂	2.519	0.84	8:4
$\alpha\text{-Al}_2\text{O}_3$	4.172	0.83	6:4

general lattice energy equation that can be applied to any crystal regardless of the crystal structure

$$U = \frac{1200.5VZ_+Z_-}{r_c + r_a} \left(1 - \frac{0.345}{r_c + r_a} \right) \text{ kJ mol}^{-1}$$

Most important advantage of the Kapustinskii equation:

It is possible to apply the equation for lattice calculations of crystals with polyatomic ions (e.g. KNO₃, (NH₄)₂SO₄ ...).

A set of „thermochanical radii“ was derived for further calculations of lattice enthalpies

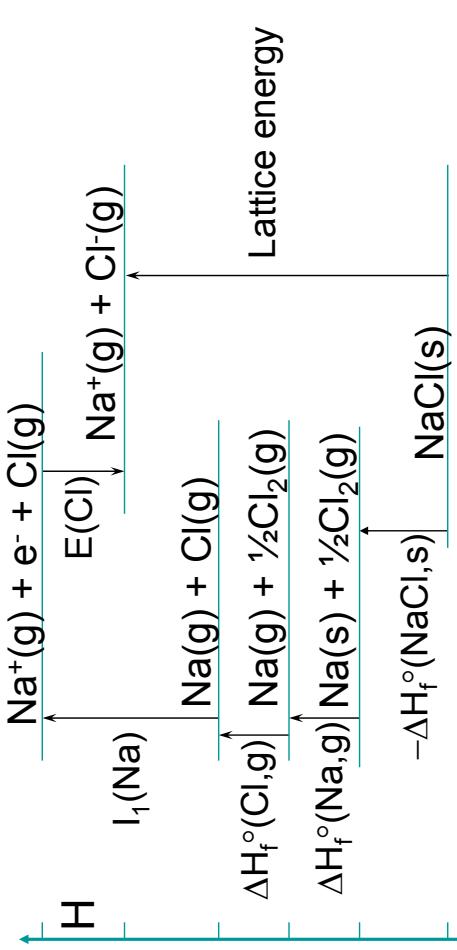
Table 1.13 Thermochanical radii of polyatomic ions*

Ion	pm	Ion	pm	Ion	pm
NH ₄ ⁺	151	ClO ₄ ⁻	226	MnO ₄ ²⁻	215
Me ₂ N ⁺	215	CN ⁻	177	O ₂ ²⁻	144
PH ₃ ⁺	171	CNS ⁻	199	OH ⁻	119
AlCl ₄ ⁻	281	CO ₃ ²⁻	164	PF ₆ ⁻	282
BF ₄ ⁻	218	IO ₃ ⁻	108	PCl ₆ ⁻	299
BH ₇ ⁻	179	N ₃ ⁻	181	PBf ₆ ⁻	328
BrO ₃ ⁻	140	NCO ⁻	189	PF ₆ ⁻	328
CH ₃ COO ⁻	148	NO ₂ ⁻	178	SO ₄ ²⁻	244
ClO ₃ ⁻	157	NO ₃ ⁻	165	SeO ₄ ²⁻	235

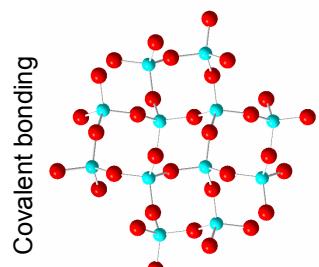
*J.E. Huheey (1983) *Inorganic Chemistry*, 3rd edn, Harper and Row, London, based on data from H.D.B. Jenkins and K.P. Thakur (1979) *J. Chem. Ed.*, **56**, 576.

Born Haber cycle

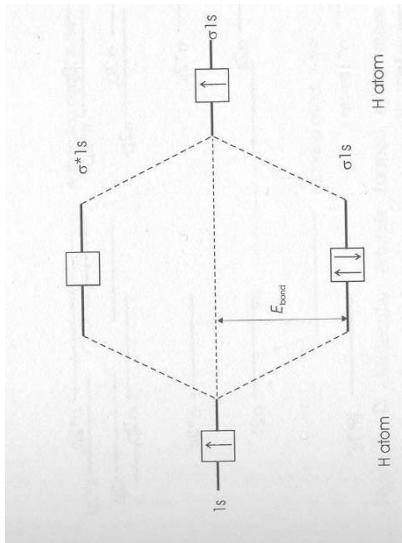
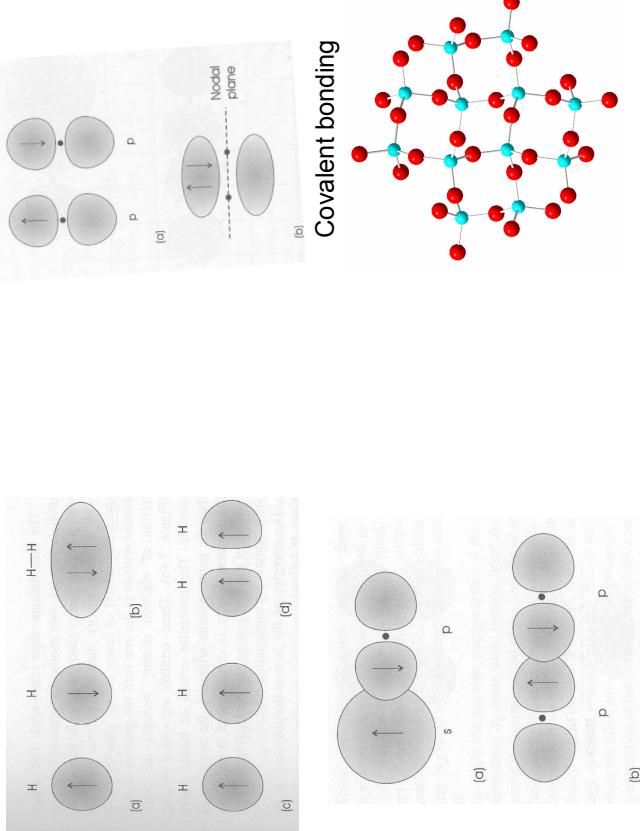
The lattice energy can be calculated by using Hess law and the following scheme:



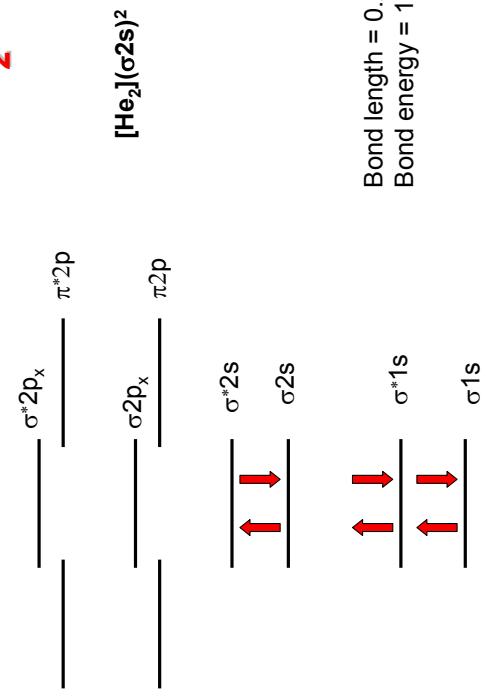
Crystalline materials show a range of different bond types:



Covalent bonding is a result of orbital overlaps
Spins in overlapping orbitals must be antiparallel

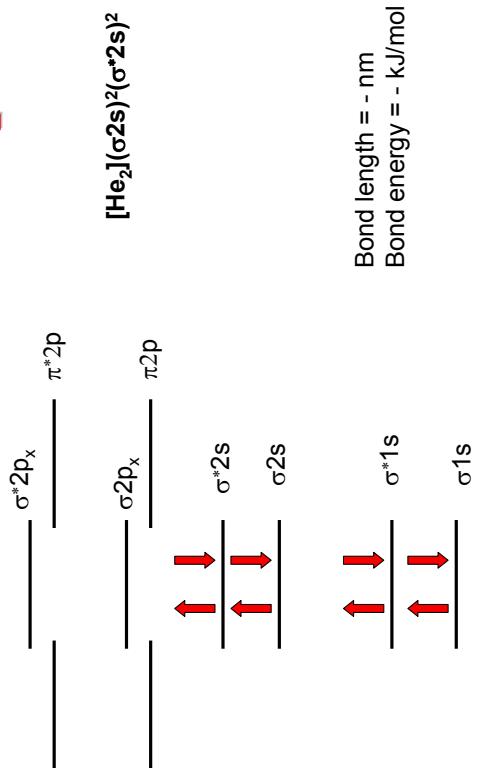


Li₂



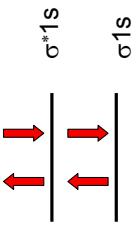
Bond length = 0.267 nm
 Bond energy = 101 kJ/mol

Be₂

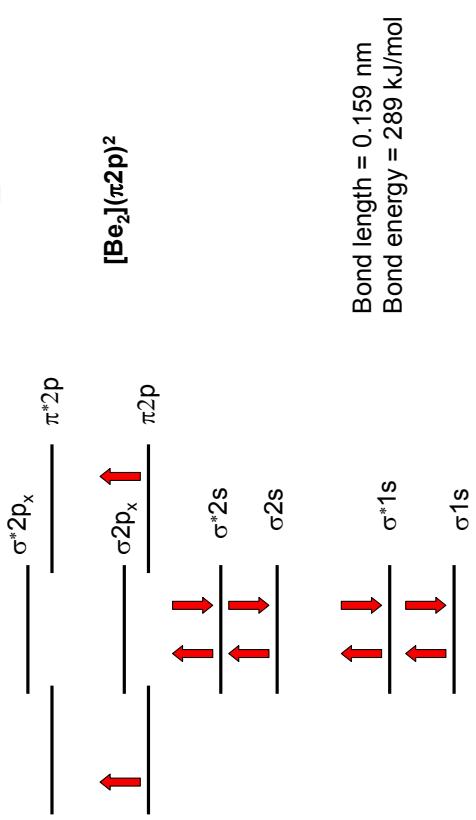


Bond length = - nm
 Bond energy = - kJ/mol

σ1s

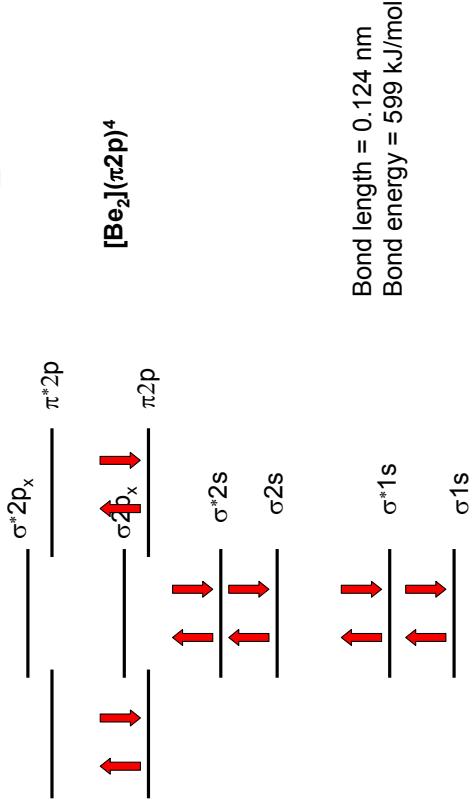


B₂



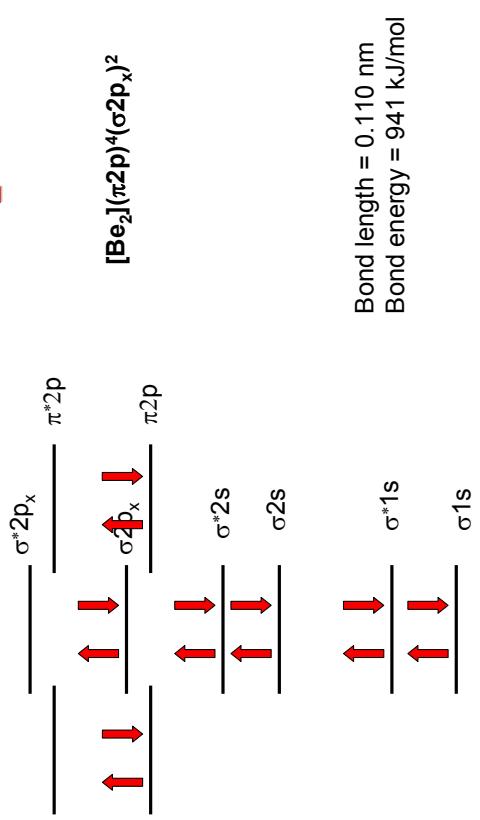
Bond length = 0.159 nm
Bond energy = 289 kJ/mol

C₂



Bond length = 0.124 nm
Bond energy = 599 kJ/mol

N₂

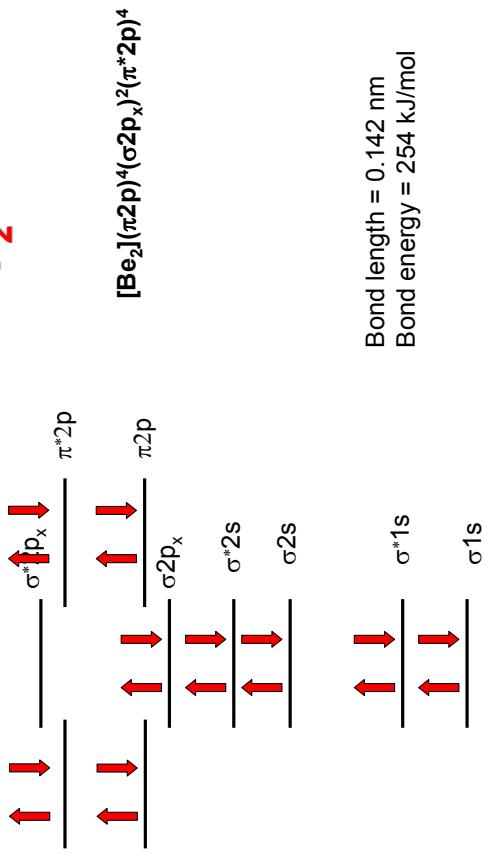


Bond length = 0.110 nm
Bond energy = 941 kJ/mol

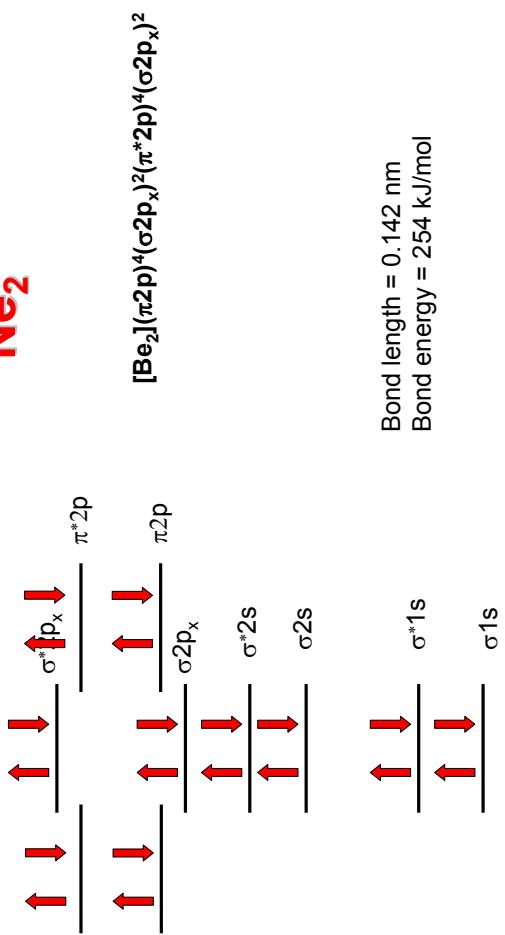
O₂

Bond length = 0.121 nm
Bond energy = 494 kJ/mol

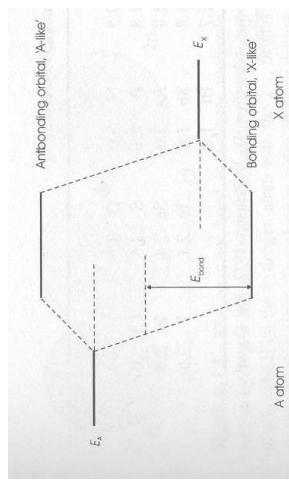
F₂



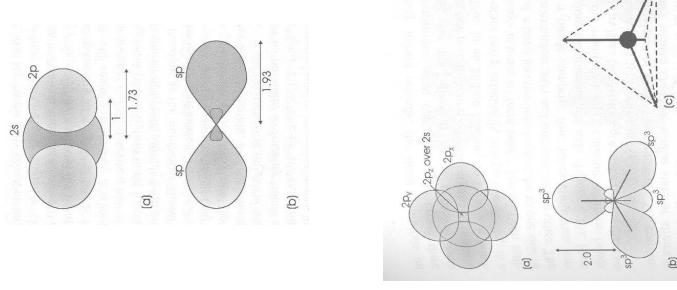
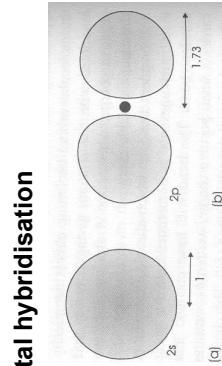
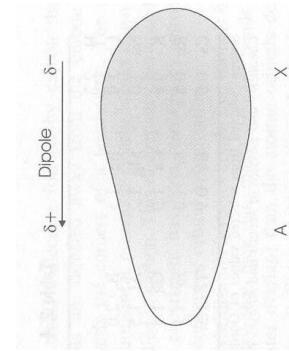
Ne₂



Bonding between unlike atoms



Orbital hybridisation



Multiple bonds

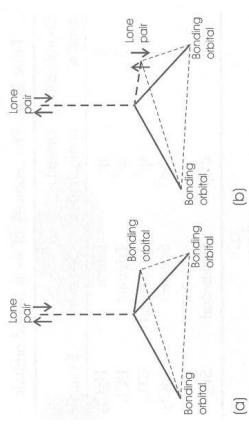
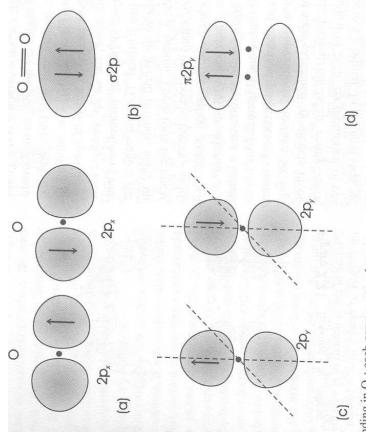
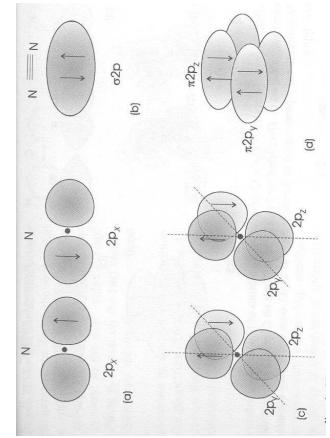
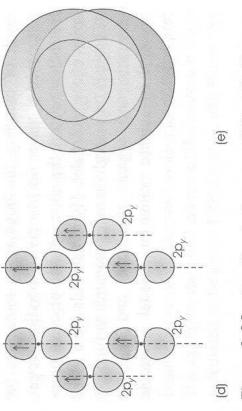
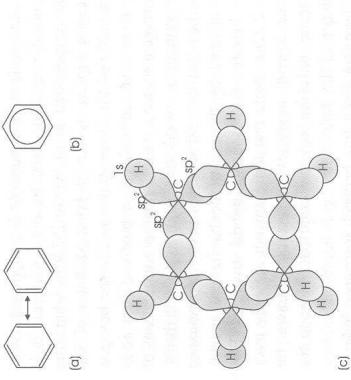


Table 2.5 The geometry of some hybrid orbitals

Coordination number	Orbital configuration	Geometry	Example
2	sp	Linear	HgCl ₂
3	sp ²	Trigonal	BCl ₃
4	sp ³	Tetrahedral	CH ₄
	dsp ²	Square planar	PdCl ₂
6	d ² sp ³	Octahedral	SF ₆



Resonance



Covalent bonding occurs when the outer electronic charge density on an anion is polarized towards and by a neighbouring cation.

The net effect is electron density between the atoms.

Partial covalent bonding

SrO NaCl-type
BaO NaCl-type
HgO linear O-Hg-O segments

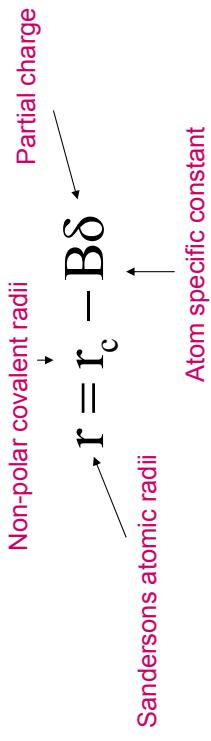
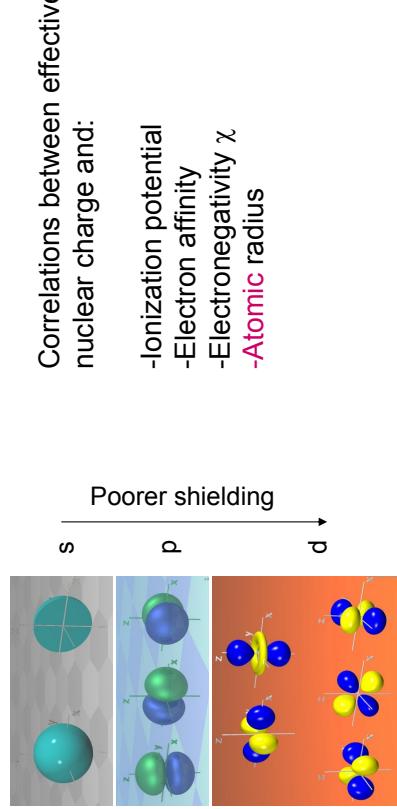
AlF₃ ReO₃ related ionic solid
AlCl₃ Layered polymeric structure
AlBr₃ Molecular Al₂Br₆ dimer
AlI₃ Molecular Al₂I₆ dimer

Increasing difference in electronegativity

Sandersons model

The properties of solids is much dependent on how valence electrons 'feel the situation'.

Effective nuclear charge



The partial charges can not be measured directly but estimated from Sandersons electronegativity scale.

Sandersons model – Electronegativity

Sanderson developed a new electronegativity table (S) where:

$$S = \frac{D}{D_a}$$

Electron density derived from linear interpolation of inert gas elements.

Principle of electronegativity equalization

$$S_b = \sqrt{S_{Na} S_F} = 2.006$$

The electronegativity is divided between both atoms in the bond

Table 2.10 Some electronegativity and size parameters of atoms. (After Sanderson 1975)

Element	S^*	r_c (Å) [†]	B (solid) [‡]	ΔS_c §	r_i (Å) [¶]
H	3.55	0.32	0.92	3.92	
Li	0.74	1.34	0.812	1.77	0.53
Be	1.99	0.91	0.330	2.93	0.58
B	2.93	0.82		2.56	
C	3.79	0.77	4.05	4.05	
N	4.49	0.74	4.41	4.41	
O	5.21	0.70	4.01	4.75	1.10
F	5.75	0.68	0.925	4.99	1.61
Na	0.70	1.34	0.763	1.74	0.78
Mg	1.56	1.38	0.349	2.60	1.03
Al	2.22	1.26		3.10	
Si	2.84	1.17	3.51	3.51	
P	3.43	1.10	3.85	3.85	
S	4.12	1.04	0.657	4.22	1.70
Cl	4.93	0.99	1.191	4.62	2.18
K	0.42	1.96	0.956	1.35	1.00
Ca	1.22	1.74	0.550	2.30	1.19
Zn	2.98			3.58	
Ga	3.28			3.77	
Ge	3.59	1.22	3.94	3.94	
As	3.90	1.19	4.11	4.11	
Se	4.21	1.16	0.665	4.27	1.83
Br	4.53	1.14	1.242	4.43	2.38
Rb	0.36	2.16	1.039	1.25	1.12
Sr	1.06	1.91	0.429	2.14	1.48
Ag	2.59	1.50	0.208	2.14	1.48
Cd	2.84	1.46	0.132	3.35	1.33
Sn	3.09	1.40		3.16	3.66
Sb	3.34	1.38		3.80	
Te	3.59	1.35		3.94	2.04
I	3.84	1.33		4.08	2.71
Cs	0.28	2.35		3.93	1.39
Ba	0.78	1.98		1.93	1.63
Hg	2.93			3.59	2.85
Tl	3.02	1.48		3.21	3.69
Pb	3.08	1.47		3.74	
Bi	3.16	1.46			

* Sanderson's electronegativity.

† Non-polar covalent radii.

‡ Constant in equation (2.29) relating size to partial charge.

§ Change in electronegativity on acquiring unit charge.

¶ Radii calculated for singly charged ions.

Sandersons model – Partial charge

Partial charge is the ratio of change in electronegativity undergone by an atom on bond formation to the change it would have undergone on becoming completely ionic with charge + or -1

A point of reference is necessary.
The bonds in NaF is 75% ionic.

The change in electronegativity on acquiring a + or -1 charge is ΔS_c :

$$\Delta S_c = 2.08\sqrt{S}$$

Partial charge can then be defined as:

$$\delta = \frac{\Delta S}{\Delta S_c} \quad \Delta S = S - S_b$$

Example: BaI₂

$$\begin{array}{ll} S_{Ba} = 0.78 & \Delta S_c^{Ba} = 1.93 \\ S_I = 3.84 & \Delta S_c^I = 4.08 \end{array}$$

$$Sb = \sqrt[3]{(0.78 * 3.84 * 3.84)} = 2.26$$

$$\begin{array}{ll} \Delta S_{Ba} = 2.26 - 0.78 = 1.48 \\ \Delta S_I = -3.84 + 2.26 = -1.58 \end{array}$$

$$\delta = \frac{\Delta S_{Ba}}{\Delta S_c^{Ba}} = \frac{1.48}{1.93} = 0.78$$

$$\delta = \frac{\Delta S_I}{\Delta S_c^I} = -\frac{1.58}{4.08} = -0.39 \quad | -0.39$$

Atomic radius can be calculated from: $r = r_c - B\delta$

	Ba	I
r _c	1.98 Å	1.33 Å
B	0.348	1.384

$$\begin{array}{ll} r_{Ba} = 1.98 \text{ Å} - 0.348 * 0.78 \text{ Å} = 1.71 \text{ Å} \\ r_I = 1.33 \text{ Å} + 1.384 * 0.39 \text{ Å} = 1.87 \text{ Å} \\ \text{Ba-I} = 3.58 \text{ Å}, \text{ Observed: } 3.59 \text{ Å!} \end{array}$$

Mooser – Pearson plots and ionicities

The radius ratio is unsatisfactory in prediction of structure type.

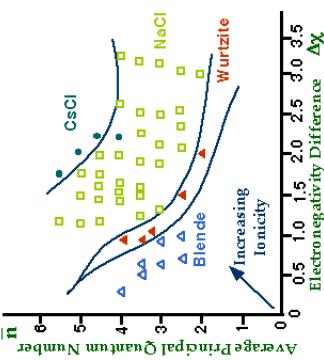
Alternative: Plot the -

Average principal quantum number vs. Difference in electronegativity

Mooser-Pearson Plot for AB

Huger ions with large Dx gives ionic structures.

Smaller atoms tend to have directional bonding.



Bond valence and bond length

Valence bond theory is easily adaptable to molecules.

An approach for crystalline solids is to introduce **bond valence (bv)**.

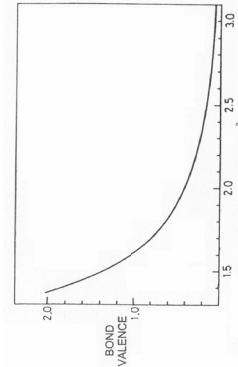
The sum of **bv** for all bonds to an atom must equal the **valence** of that atom. → **the valence sum rule.**

$$V_i = \sum_j b_{ij} v_j$$

The **bv** is found empirically from other structures and vary with bond length

Each bond is treated as an individual

$$b_{ij} = \left(\frac{R_0}{R} \right)^N$$

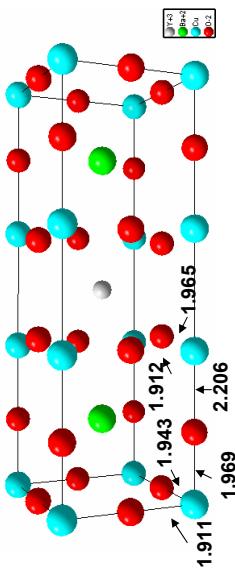


Bond valence and bond length

Bond valence and bond length, example YBCO

The bond valence scheme can be used to:

- Check the correctness of a proposed structure.
- Locate hydrogen atoms for structures determined by X-ray diffraction
- Distinguish between Al^{3+} and Si^{4+} in aluminosilicates.



What is the oxidation state for Cu?

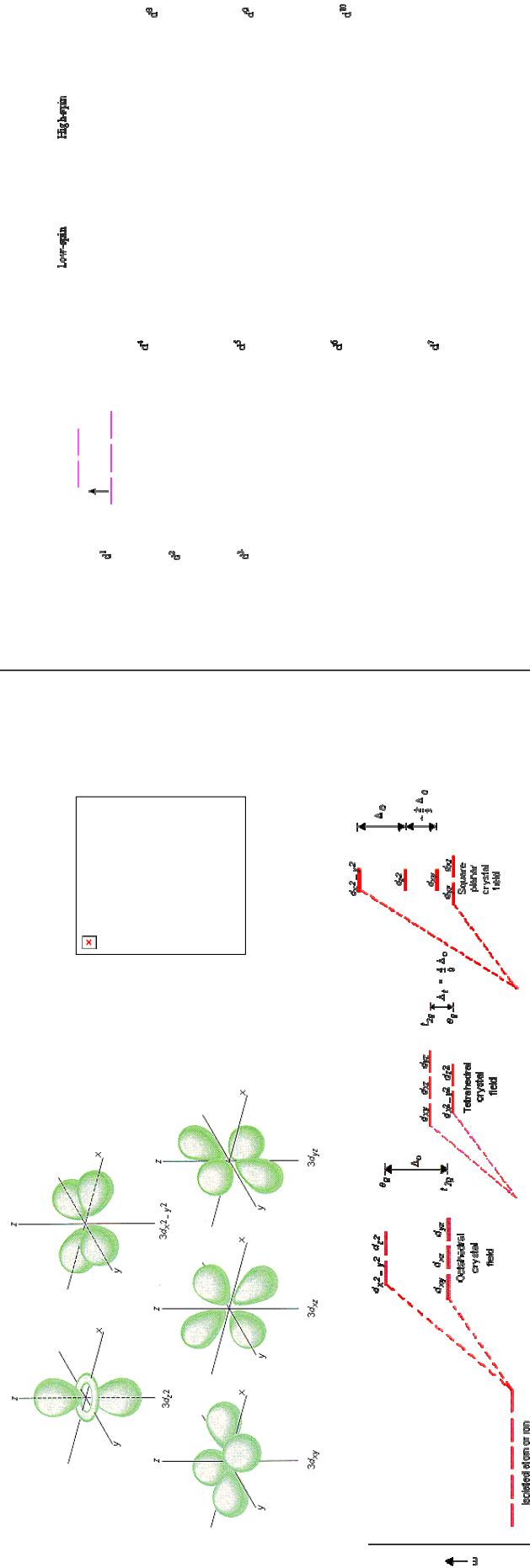
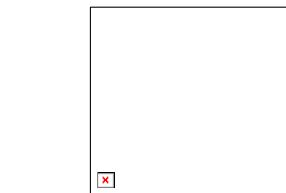
$$V_{\text{Cu}} = \sum_{j=1}^6 b_{\text{Cu},j} = \sum_{j=1}^6 c \left(\frac{R_0}{R_{\text{Cu},j}} \right)^N$$

For Cu:
 $c = 0.333$; $R_0 = 206.8 \text{ pm}$; $N = 5.4$

$$\begin{aligned} V_{\text{Cu}(1)} &= 2 * 0.510 + 2 * 0.466 + 2 * 0.434 = 2.82 \\ V_{\text{Cu}(2)} &= 2 * 0.509 + 2 * 0.439 + 1 * 0.235 = 2.13 \end{aligned}$$

$\text{Y}^{3+}\text{Ba}^{2+}_2\text{Cu}^{2+}_3\text{O}_7$ leads to a Cu average of 2.33

Non-bonding electron effects, Crystal field theory

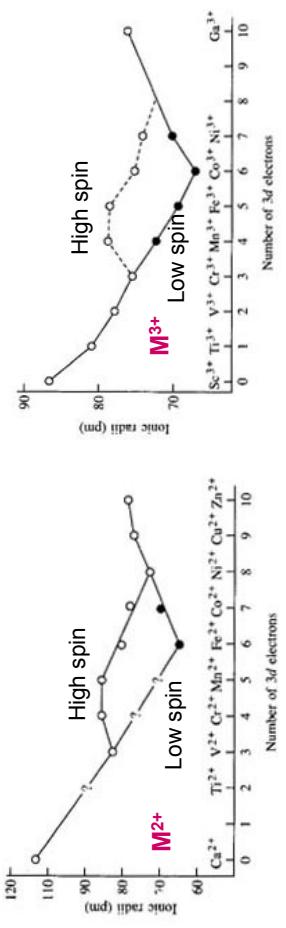


Non-bonding electron effects, Crystal field theory

Non-bonding electron effects, Crystal field theory

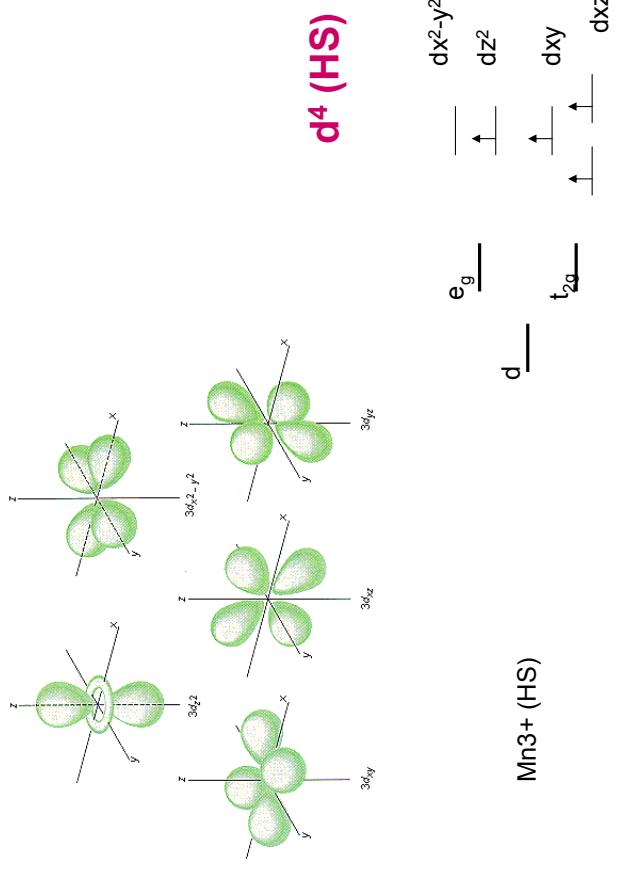
Ionic Radii. For a given oxidation state, the ionic radius decreases steadily on going from left to right in a transition series.

Populating antibonding orbitals (i.e. filling the e_g levels in an octahedron) leads to an increase in ionic radius and to weaker bonds. Therefore, the ionic radius depends on the spin state of the metal (i.e. high spin or low spin).



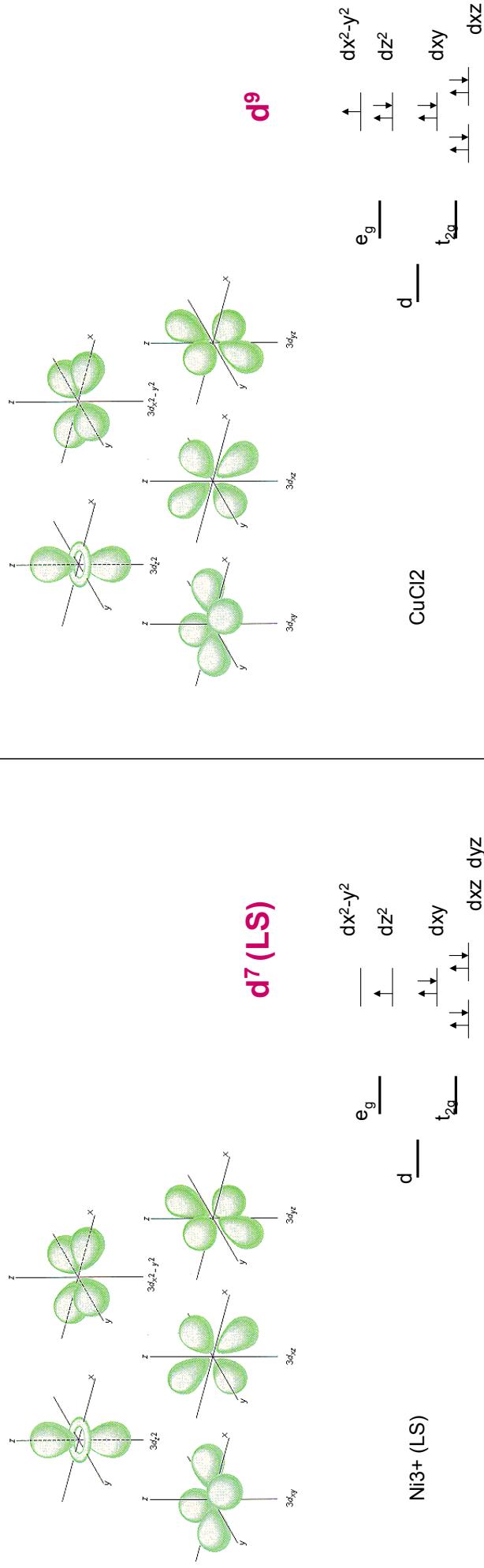
Non-bonding electron effects, Jahn-Teller

d⁴ (HS)



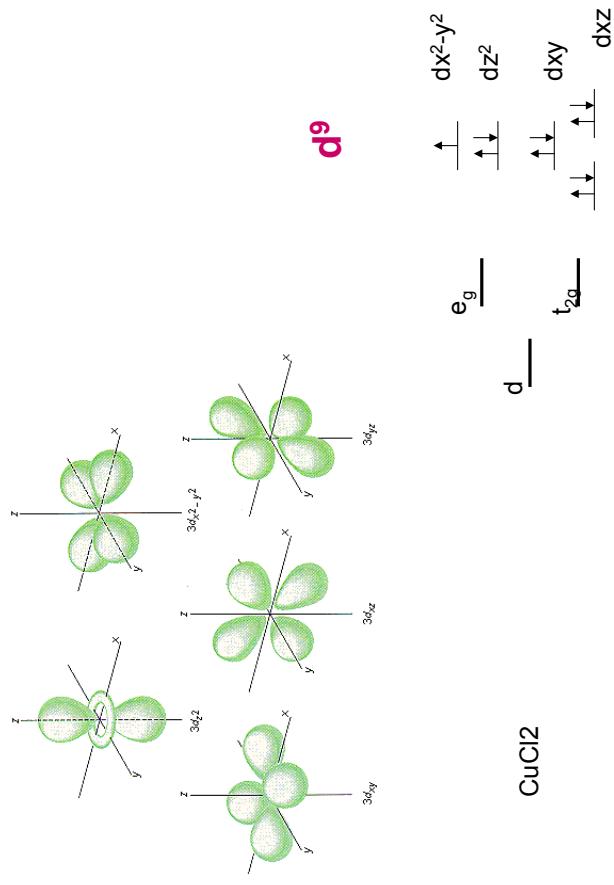
Non-bonding electron effects, Jahn-Teller

d⁷ (LS)



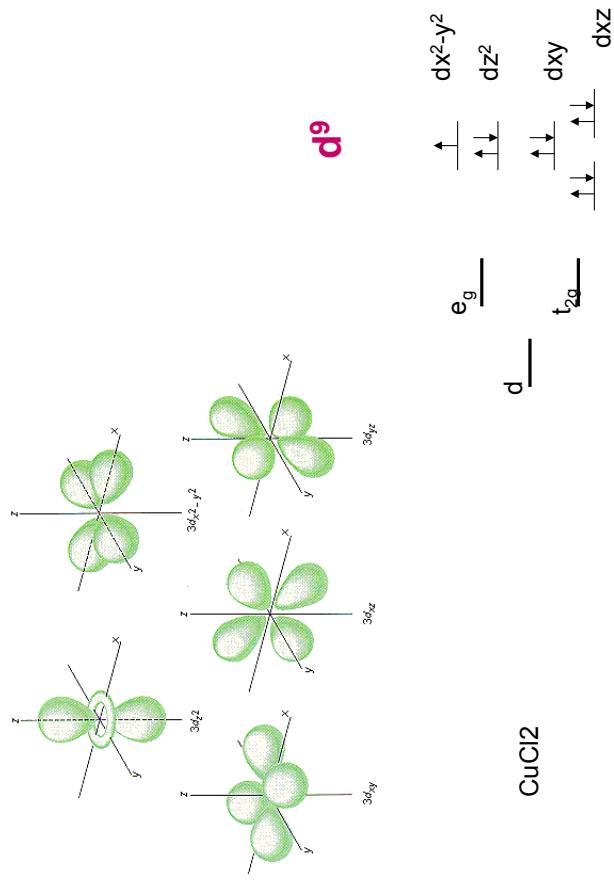
Non-bonding electron effects, Jahn-Teller

d⁹

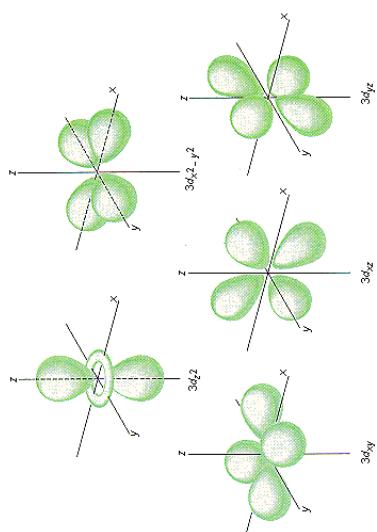


Non-bonding electron effects, Jahn-Teller

Ni3+ (LS)



Non-bonding electron effects, d⁸



Crystal field stabilization

Low - spin
High - spin

$\Delta_{\text{cryst.field}} > P$
 $\Delta_{\text{cryst.field}} < P$

$\Delta(5d) > \Delta(4d) > \Delta(3d)$
high spin
low spin

Octahedral field: Crystal field stabilization

$$= \frac{\Delta}{10} [4n_{t2g} - 6n_{eg}]$$

Example:
 V^{2+} : $\overline{\quad \quad \quad \quad \quad}$ $\Delta \text{ CFSE} = \Delta/10[4*3-0] = 1.2\Delta$
 Mn^{2+} (hs): $\overline{\quad \quad \quad \quad \quad}$ $\Delta \text{ CFSE} = \Delta/10[4*3-6*2] = 0$

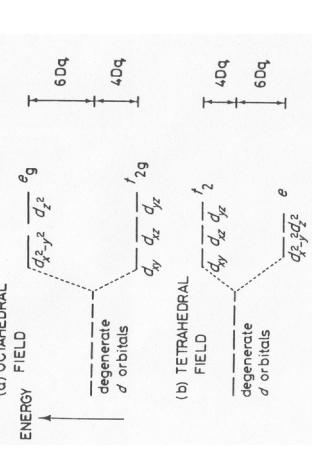
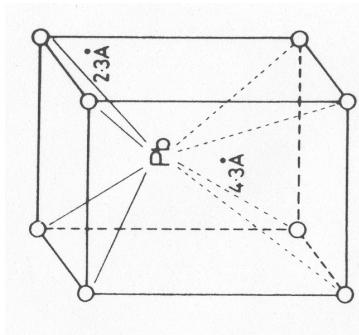
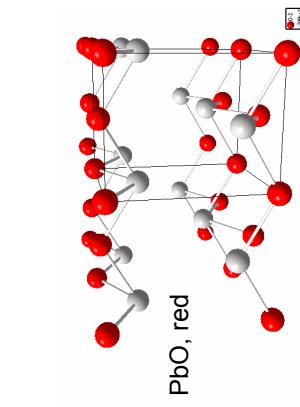


Table 2.14 Crystal field stabilization energies (kJ mol^{-1}) estimated for transition metal oxides. (Data from Dunlap, <i>Adv. Inorg. Org. Chem.</i> , 2, 1-60, 1960)			
Ion	Octahedral stabilization	Tetrahedral stabilization	Excess octahedral stabilization
Tl ³⁺	d ¹	87.4	58.5
V ³⁺	d ²	160.4	106.6
Cr ³⁺	d ³	245.5	66.9
Mn ³⁺	d ⁴	135.4	40.1
Fe ³⁺	d ⁵	0	0
Mn ²⁺	d ⁶	0	0
Fe ²⁺	d ⁶	49.7	35.0
Co ²⁺	d ⁷	92.8	61.9
Ni ²⁺	d ⁸	122.1	35.9
Cd ²⁺	d ⁹	90.3	26.8

Factors determining the structure

Size
 r_M/r_X
 $\text{CN} = 2, 4, 6, 8, \dots$

Bonding type
Ionic
Covalent
Metallic



Non-bonding electron effects, Inert pair effect

Heavy p-group elements (Tl, Sn, Pb, Sb) commonly show valence two less than group valence.

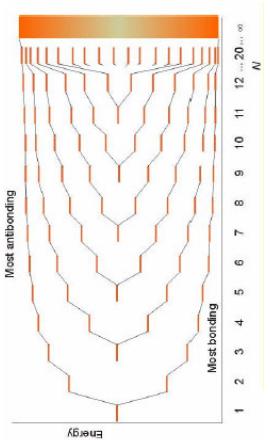
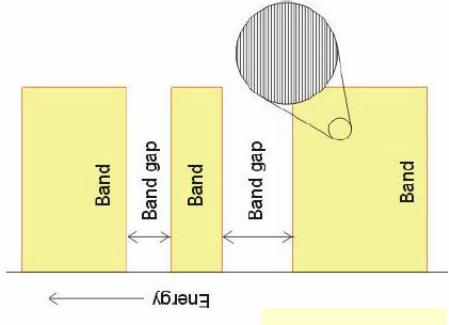
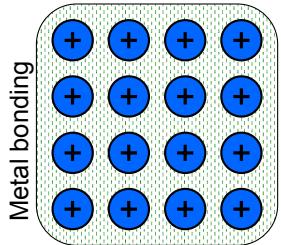
Size	r_M/r_X	$\text{CN} = 2, 4, 6, 8, \dots$	Bonding type	Ionic	Covalent	Metallic
MX	1		Directional dependant bonds, hybridization			dense packed structures
MX ₂	2					tetr. holes
MX ₃	3					oct. holes
..				trig. holes
MX _n	n					trig. bipyr. polyhedra

energetical ordering

Electronical effects such as 'lone pairs', ligand field stabilizations

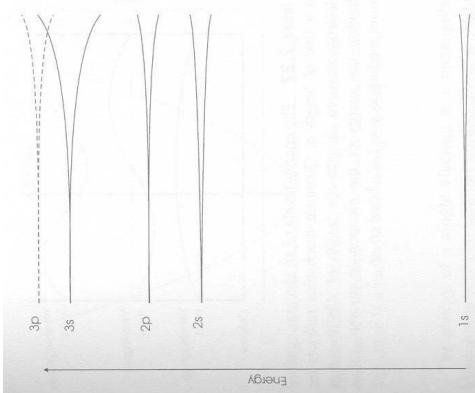
Crystalline materials show a range of different bond types:

Metallic bonding



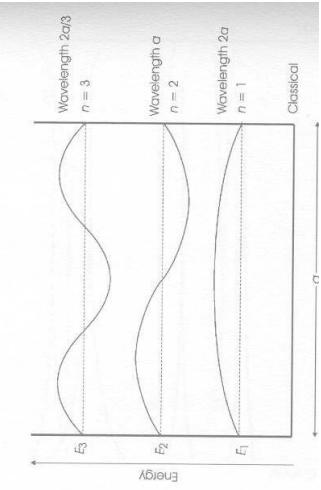
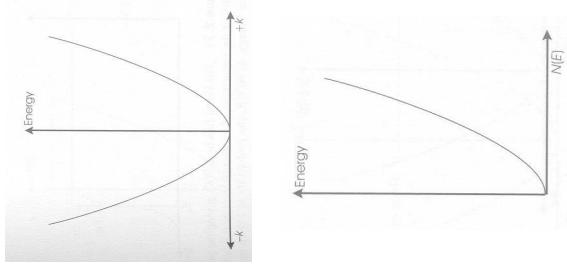
Broadening of atomic orbital into energy bands

The broadening is dependent on the orbital overlap

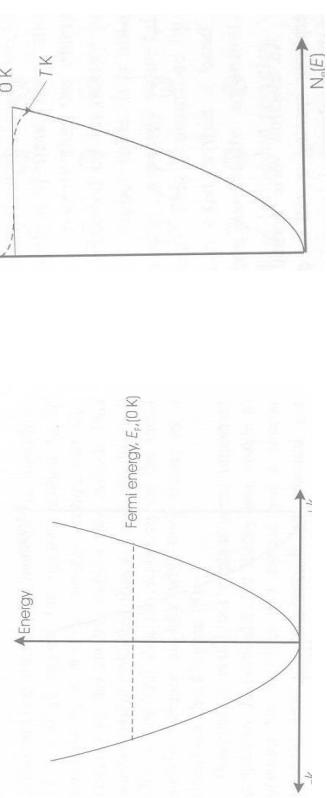


Free electron gas

The electrons are free to move through the solid as if it were an electron gas in a container determined by the outer perimeter of the solid.

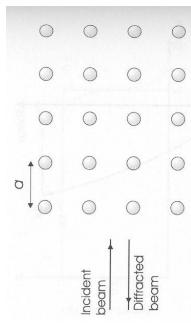
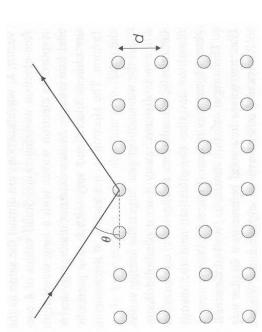
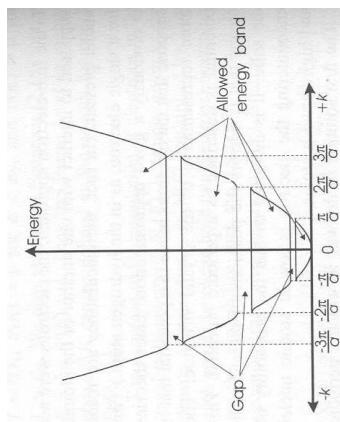


The Fermi energy



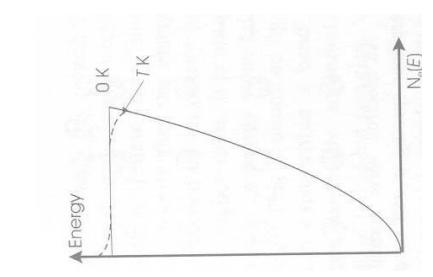
$$n\lambda = 2d \sin \theta = \frac{\pi n}{2k}$$

$$\mathbf{k} = \pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}, \dots$$



Brillouin zones

Moving electrons do interact with the atomic nucleus!



Bands in ionic and covalent solids

